

dx₂ p_y

Derivation of the equilibrium of the consumer:-

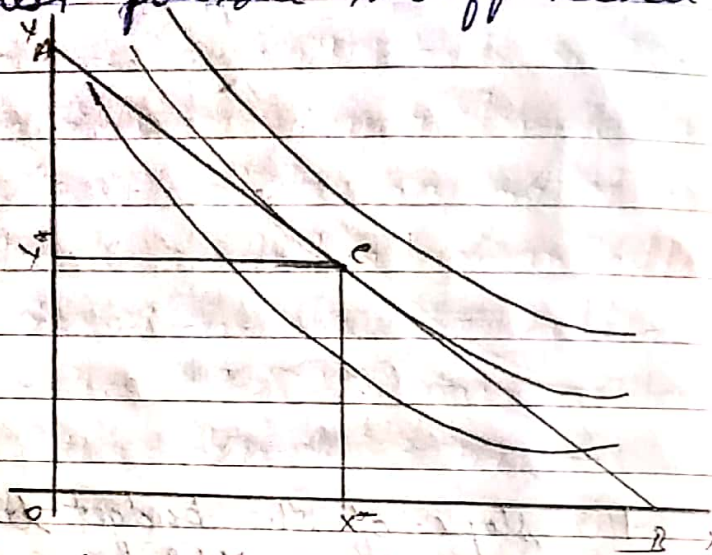
The consumer is in equilibrium when he maximizes his utility, given income and the market prices. Two conditions must be fulfilled for the consumer to be in equilibrium.

① The MRS ~~xxx~~ be equal to the ratio of market prices. $MRS_{xy} = \frac{MU_x}{MU_y} = \frac{P_x}{P_y}$

② The indifference curve be convex to the origin. This condition is fulfilled by the axiom of diminishing MRS_{xy}, which states that the slope of the indifference curve decreases as we move along the curve from the left downward to right.

KeyNotes
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Graphical presentation of the equilibrium of consumer - Given the I.D. map of the consumer and his budget line, the equilibrium is defined by the point of tangency of the budget line with the highest possible indifference curve.



at the point (C) of tangency the slope of the budget line (P_x/P_y) and of the indifference curve ($MRS_{xy} = MU_x/MU_y$) are equal

$$\frac{MU_x}{MU_y} = \frac{P_x}{P_y} = MRS_{xy}$$

at point C both the conditions are fulfilled and the consumer maximizes his utility by buying x^* and y^* of the two commodities.

Mathematical derivation of the equilibrium:

Maximize $U = f(q_1, q_2, \dots, q_n)$

Subject to $\sum_{i=1}^n q_i P_i = q_1 P_1 + q_2 P_2 + \dots + q_n P_n = Y$

Solution: The composite function

$$\Phi = U - \lambda (q_1 P_1 + q_2 P_2 + \dots + q_n P_n - Y)$$

The first order condition the maximization of a function is equal to zero.

Differentiating Φ with respect to $q_1, \dots, q_n$ and λ and equating to zero.

$$\frac{\partial Q}{\partial a_1} = \frac{\partial U}{\partial a_1} - \lambda (P_1) = 0 \quad \text{--- (i)}$$

$$\frac{\partial Q}{\partial a_2} = \frac{\partial U}{\partial a_2} - \lambda (P_2) = 0 \quad \text{--- (ii)}$$

$$\frac{\partial Q}{\partial a_n} = \frac{\partial U}{\partial a_n} - \lambda (P_n) = 0 \quad \text{--- (iii)}$$

$$\frac{\partial Q}{\partial \lambda} = - (a_1 P_1 + a_2 P_2 + \dots + a_n P_n - Y) = 0 \quad \text{--- (iv)}$$

from these equation we obtain

$$\frac{\partial U}{\partial a_1} = \lambda P_1, \quad \frac{\partial U}{\partial a_2} = \lambda P_2, \quad \frac{\partial U}{\partial a_n} = \lambda P_n$$

Substituting and solving for λ

$$\lambda = \frac{MU_1}{P_1} = \frac{MU_2}{P_2} = \dots = \frac{MU_n}{P_n}$$

and if commodity x and commodity y

$$\frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

we observe that the equilibrium condition are identical in the cardinalist approach and in indifference curve approach.

$$\frac{MU_1}{P_1} = \frac{MU_2}{P_2} = \frac{MU_x}{P_x} = \frac{MU_y}{P_y} = \dots = \frac{MU_n}{P_n}$$