

The indifference-curve Theory

Assumption:-

- (i) Rationality :- The consumer is assumed to be rational - he aims at the maximization of his utility, given income and market prices, and full knowledge of all relevant information.
- (ii) Utility is ordinal :- The consumer can rank his preferences (order the various 'baskets of goods') according to the satisfaction of each basket. He need not know precisely the amount of satisfaction.
- (iii) Diminishing marginal rate of substitution :- Indifference curve to be convex to origin and the slope of the Indifference curves increases. The slope of the Indifference curve is called the ~~slope of the~~ marginal rate of substitution of the commodities. So the I.C. curve theory based on Diminishing marginal rate of substitution.

⑩ The total utility of the consumer depends upon the quantities of the commodities consumed.

$$U = f(q_1, q_2, q_3, \dots, q_n)$$

⑪ Consistency: The consumer is consistent in his choice that is if one period he chooses bundle A over B, he will not choose B over A.

If $A \succ B$ then $B \not\succ A$ in any state.

⑫ Transitivity of choice: In transitivity assumption if consumer preferred bundle A to B and B preferred to C.

If $A \succ B$ and $B \succ C$ then $A \succ C$

Equilibrium of the consumer:

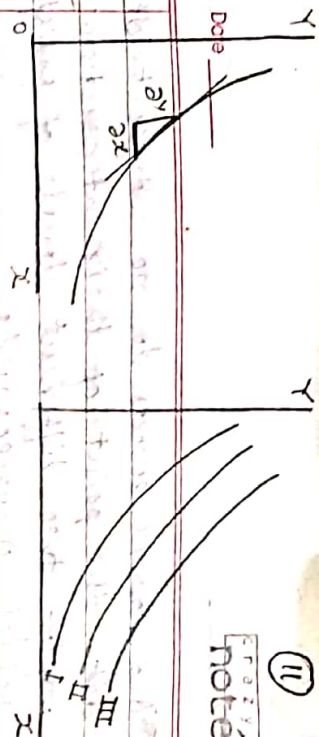
Equilibrium of the consumer is the concept of indifference curves and their slope (MRS) and the concept of budget line.

Indifference curve:

An indifference curve is the locus of points - particular combinations or bundles of goods - which yield the same utility to the consumer.

The indifference map shows all the ~~indifference~~ indifference curves which show the preferences of the consumers. Higher indifference curve yield higher level of satisfaction and are preferred.

and the combinations of goods on a lower indifference curve yield a lower utility.



⑪

notes

It is assumed that the commodities Y and X can substitute one another to a certain extent but are not perfect substitutes. The negative of the slope of an indifference curve at any one point is called the marginal rate of substitution of the two commodities X and Y.

$$\text{Slope of I.D curve} = -\frac{dY}{dX} = MRS_{XY}$$

The MRS of X for Y is defined as the number of units of commodity Y that must be given up in exchange for an extra unit of commodity X so that consumer maintain that same level of satisfaction.

The marginal rate of substitution is equal to the ratio of the marginal utilities of the commodities involved in the utility function.

$$MRS_{XY} = \frac{MU_X}{MU_Y} \quad \text{or} \quad MRS_{YX} = \frac{MU_Y}{MU_X}$$

Properties of the indifference curve:

① The indifference curve has a negative slope which denotes that if the quantity of one commodity (Y) decreases, the quantity of other (X) must increase.

② The indifference curve lies, the higher the level of utility it denotes, bundles of goods on a higher indifference curve are preferred by the rational consumer.

Indifference curves do not intersect. At any point, the point of their intersection would imply two different levels of satisfaction which is impossible.

Mathematical explanation:
 $U = f(x, y)$

The equation of an indifference curve is $U = f(x, y) = k$

where k is constant. The total differential

$$du = \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy = 0$$

$$du = MU_x dx + MU_y dy = 0$$

$$MU_x dx - \frac{dx}{dy} = \frac{MU_x}{MU_y}, \text{ then } \frac{dx}{dy} = \frac{MU_y}{MU_x}$$

The indifference curves are convex to origin and indifference curve moves along the curve from the left downwards to right. The MRS of the commodities is diminishing.

The budget constraint of the consumer:

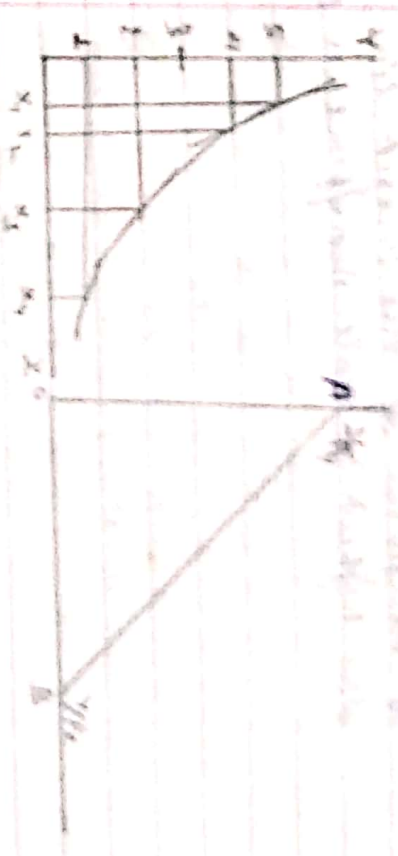
The consumer has a given income I and sets limits to his maximising behaviour. The income constraint, in case of two commodities

$$Y = P_x X + P_y Y$$

$$MY = \frac{Y}{P_y} - \frac{P_x}{P_y} X$$

Suppose if $Y_x = 0$ the consumer can buy $\frac{Y}{P_y}$ units of Y and if $Y_y = 0$ the consumer can buy $\frac{Y}{P_x}$ units of X . In these result are shown by points A

and B. If we give these points with a line we obtain the budget line whose slope is the ratio of the prices of the two commodities



The slope of the budget line

$$\frac{OA}{OB} = \frac{Y/P_y}{X/P_x} = \frac{P_x}{P_y}$$

Mathematically, the slope of the budget line is the derivative $\frac{dx}{dy} = \frac{P_y}{P_x}$

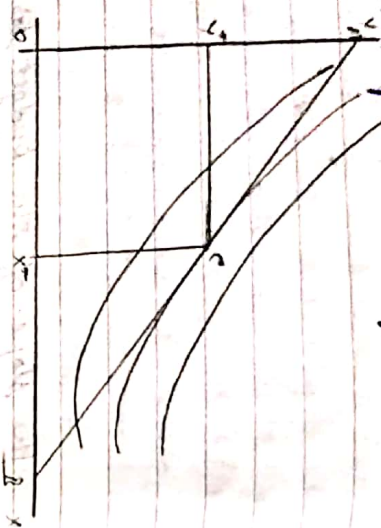
Derivation of the equilibrium of the consumer:

The consumer is in equilibrium when his maximises his utility, given income and the market prices. These conditions must be fulfilled for the consumer to be in equilibrium

- ① The MRS must be equal to the ratio of market prices. $MRS_{xy} = \frac{MU_x}{MU_y} = \frac{P_x}{P_y}$
- ② The indifference curve be convex to origin. This condition is fulfilled by the axiom of diminishing MRS, which states that the slope of the indifference curve decreases as we move along the curve from the left downwards to right.

Graphical presentation of the equilibrium of a consumer

Given the I.D. map of the consumer and his budget line, the equilibrium is defined by the point of tangency of the budget line with the highest possible indifference curve.



at the point (C) of tangency the slope of the budget line (P_x/P_y) and of the indifference curve ($MU_x = MU_x/MU_y$) are equal

$$\frac{MU_x}{MU_y} = \frac{P_x}{P_y} = MRS_{xy}$$

at point C both the condition are fulfilled and the consumer maximizes his utility by buying x^* and y^* of the two commodities.

Mathematical derivation of the equilibrium:

Maximize $U = f(q_1, q_2, \dots, q_n)$
Subject to $\sum_{i=1}^n q_i p_i = q_1 p_1 + q_2 p_2 + \dots + q_n p_n = Y$

Substitution: The composite function

$$Q = U - \lambda (q_1 p_1 + q_2 p_2 + \dots + q_n p_n - Y)$$

The first order condition on the maximization of a function is always set to zero.

KeyNotes: Differentiating Q with respect to q_1 , q_n and λ and equating to zero.

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$$\frac{\partial Q}{\partial q_1} = \frac{\partial U}{\partial q_1} - \lambda (P_1) = 0 \quad \text{--- (I)}$$

$$\frac{\partial Q}{\partial q_2} = \frac{\partial U}{\partial q_2} - \lambda (P_2) = 0 \quad \text{--- (II)}$$

$$\frac{\partial Q}{\partial q_n} = \frac{\partial U}{\partial q_n} - \lambda (P_n) = 0 \quad \text{--- (III)}$$

$$\frac{\partial Q}{\partial \lambda} = -(q_1 p_1 + q_2 p_2 + \dots + q_n p_n - Y) = 0 \quad \text{--- (IV)}$$

from these equation we obtain

$$\frac{\partial U}{\partial q_1} = \lambda P_1, \quad \frac{\partial U}{\partial q_2} = \lambda P_2, \quad \frac{\partial U}{\partial q_n} = \lambda P_n$$

Substituting and solving for λ

$$\lambda = \frac{MU_1}{P_1} = \frac{MU_2}{P_2} = \dots = \frac{MU_n}{P_n}$$

and if commodity x and commodity y

$$\frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

we observe that the equilibrium condition are identical in the cardinalist approach and in indifference curve approach.

$$\frac{MU_1}{P_1} = \frac{MU_2}{P_2} = \frac{MU_x}{P_x} = \frac{MU_y}{P_y} = \dots = \frac{MU_n}{P_n}$$

Derivation of the demand curve using the indifference curve approach:

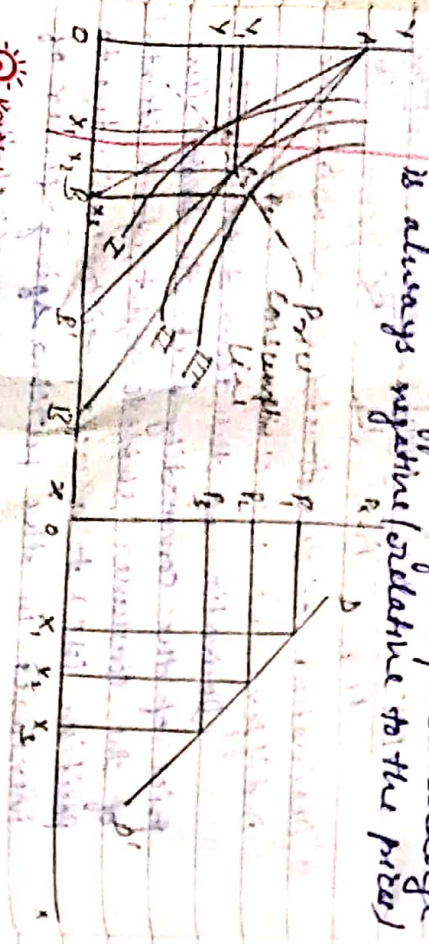
Derivation of the demand curve: The price of the commodity fall the budget line of the consumer shift to right, from its initial position (AB) to a new position (AC). due to increases in the purchasing power of

KeyNotes: the given money income of the consumer. with more purchasing power in his possession the consumer can buy ...?

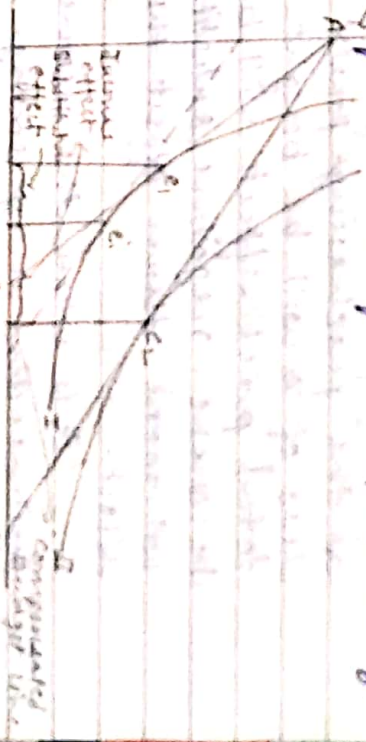
ment of a land more of y). The new budget line is tangent to a higher indifference curve. The new equilibrium occurs at the point of the original equilibrium (for y goods) showing that as price falls money the commodity will be bought. If the price of x falls continuously and we join the point of tangency of successive budget lines and higher indifference curves we form the so called price consumption line. From which we derived the demand curve for commodity x .

The demand curve for normal commodities will always have a negative slope, during the law of demand (the quantity bought increases as the price falls).

The $I-D$ curve approach the 'law of demand' is derived from Slutsky's theorem. Slutsky's theorem states that the substitution effect of a price change is always negative (relative to the price).



A fall in price of x from p_1 to p_2 results in an increase in the quantity demanded from x_1 to x_2 . This is the total price effect which may be split into two separate effects, a substitution effect and an income effect. The substitution effect is the increase in the quantity bought as the price of the commodity falls and adjusting income so as to keep the real purchasing power of the consumer. This adjustment in income is called compensating variation and is shown graphically by a parallel shift of the new budget line tangent to the initial indifference curve. The purpose of the compensating variation is to allow the consumer to remain on the same level of satisfaction as before the price change.



In the 'The compensated budget line will be tangent to the original indifference curve at point e_1 to the right of the original tangency e_1 because this line is parallel to the new budget line which is less

Step show the original one when the price of x falls. The movement from point e_1 to e_2 shows the substitution effect.