

Measure of Central value -

Types of averages -

- ① Arithmetic mean: ① Simple, and ② weighted.
- ② Median
- ③ Mode
- ④ Geometric mean
- ⑤ Harmonic mean

Arithmetic mean

Its value is obtained by adding together all the items and by dividing this total by the number of items.

① sample Arithmetic mean

$$\bar{X} = \frac{X_1 + X_2 + X_3 + \dots + X_n}{N} = \frac{\sum X}{N}$$

Short-Cut method -

$$\bar{X} = A + \frac{\sum d}{N}$$

where A is the assumed mean and d is the deviation of items from assumed mean

i.e., $d = (X - A)$

Direct Method -

$$\bar{X} = \frac{\sum fX}{N}$$

where f = Frequency; X = The variable in question; N = Total No. of observations, i.e., $\sum f$.

Short-cut Method -

$$\bar{X} = A + \frac{\sum fd}{N}$$

Calculation of Arithmetic:- Continuous Series

$$\bar{X} = \frac{\sum fm}{N}$$

where m = mid-point of various classes; f = the frequency of each class, N = the total freq...

$$\text{Mid Point} = \frac{\text{Lower Limit} + \text{Upper Limit}}{2}$$

Short-cut Method :-

$$\bar{X} = A + \frac{\sum fd}{N}$$

where A = assumed mean; d = deviations of mid-points from assumed mean i.e. $(m-A)$;
 N = total number of observations.

In order to simplify the calculations, divide the deviations by class intervals $(m-A)/i$ and then multiply by i in the formula for getting mean.

$$\bar{X} = A + \frac{\sum fd}{N} \times i$$

Calculation of Arithmetic Mean in case of open-end class -

open-end classes are those in which lower limit of the first class and the upper limit of the last class are known.

Mathematical Properties of Arithmetic Mean.

A few important mathematical properties of the arithmetic mean:-

- ① $\sum (X - \bar{X}) = 0$
- ② $\sum (X - \bar{X})^2 = \text{Minimum}$
- ③ $\bar{X} = \sum X / N$, or $N\bar{X} = \sum X$
- ④ If we have the arithmetic mean and number of items of two or more than two related groups.

$$\bar{X}_{12} = \frac{N_1 \bar{X}_1 + N_2 \bar{X}_2}{N_1 + N_2}$$

$\bar{X}_{12}$ = Combined mean of the two groups.

$\bar{X}_1$ = arithmetic mean of first group.

$\bar{X}_2$ = arithmetic mean of second group.

N_1 = number of items in the first group.

N_2 = number of items in the second group.

$$\bar{X}_{123} = \frac{N_1 \bar{X}_1 + N_2 \bar{X}_2 + N_3 \bar{X}_3}{N_1 + N_2 + N_3}$$

Weighted Arithmetic Mean:-

One of the important limitations of the arithmetic mean discussed above is that it gives equal importance of all the items. But there are cases where the relative importance of the different items is not the same. When this is so, we compute weighted arithmetic mean. The term 'weight' stands for the relative importance of the different item.

$$\bar{X}_w = \frac{\sum WX}{\sum W}$$

where $\bar{X}_w$ represents the weighted arithmetic mean; X represents the variable value.

In case of frequency distribution

$$\bar{X}_w = \frac{\sum w(fX)}{\sum w}$$

$$\bar{X}_w = \frac{w_1(f_1X_1) + w_2(f_2X_2) + \dots + w_n(f_nX_n)}{w_1 + w_2 + \dots + w_n}$$

An important problem that arises while using weighted means is regarding selection of weights.

- 1) $\bar{X} = \bar{X}_w$, if $w_1 = w_2 = \dots = w_n$
- 2) $\bar{X} < \bar{X}_w$, if $(w_2 - w_1)(X_1 - X_2) < 0$.
- 3) $\bar{X} > \bar{X}_w$, if $(w_2 - w_1)(X_1 - X_2) > 0$.

* The weighted arithmetic mean is specially useful in problem relating to:

- ① Construction of index numbers, and
- ① standardized birth and death rates.

weighted G.M

$$G.M._w = A.L. \left(\frac{N_1 \log G.M._1 + N_2 \log G.M._2}{N_1 + N_2} \right)$$